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Some Properties of Pole Solution in Six Dimensions

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Abstract
A solution with the pole configuration in six dimensions is analyzed both analytically and numerically. It is a dimensional reduction model of Radall-Sundrum type. The soliton configuration is induced by the bulk Higgs mechanism. The boundary condition is systematically solved up to the 6th order. The Riemann curvature is finite everywhere. An exact solution for the no potential case is also presented.

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1 Introduction

Seeking a realistic model of our world, from the higher-dimensional standpoint, the brane-world approach is now vigorously envestigated in various ways. It started with the wall configuration in 5 dimensions(D) spacetime[1, 2]. Basically the brane-world physics makes use of the soliton configuration. We can explore new physics, including the beyond-standard model, by making use of the distinguished properties of soliton, such as the localized zero modes, stable configuration, singularity-free behavior due to the extendedness, etc.. From the reason of improving some difficulties in 5D model[3, 4, 5], or of getting more general aspects of the brane-world physics, further-higher-dimensional models (string-like object in 6D, monopole configuration in 7D, $\cdots$) are also envestigated[6, 7, 8, 9, 10]. In this paper, we examine a 6D model.

In the original model, 3-brane(s) is introduced, by hand, as a δ-function distribution in the extra space. We do not take such approach in order to seek a non-singular solution. We induce the same configuration as a soliton (kink) solution. This situation is very similar to the relation between the Dirac and 'tHooft-Polyakov monopoles. The latter is the soliton (in the gauge+Higgs system) interpretation of the former system where the gauge potential has δ-function distribution in the plane perpendicular to the Dirac string. We seek a soliton solution in the 6D gravity + Higgs system. The main purpose is to establish the pole solution [11]. The most crucial point is to confirm the convergence of some infinite series appearing in the solution. It guarantees the boundary condition. In [11] they are solved up to the 2nd order approximation. Here we show them up to the 6th order. The result strongly shows that the series converge and the boundary conditions are sufficiently satisfied.

2 The Dimensional Reduction Model in Six Dimensions

In [11] a solution with the pole configuration in six dimensions is presented. The model is the 6D gravity coupled with the Higgs fields.

$$S[G_{AB},\Phi]=\int d^6X\sqrt{-G}(-\frac{1}{2}M^4\hat{R}-G^{AB}\partial_A\Phi^*\partial_B\Phi-V(\Phi^*,\Phi)) \quad ,$$

$$V(\Phi^*,\Phi)=\frac{\lambda}{4}(|\Phi|^2-{v_0}^2)^2+\Lambda \quad , \qquad (1)$$

where $(X^A) \equiv (x^\mu, \rho, \varphi), \mu = 0, 1, 2, 3$. x^μ's are regarded as our world coordinates, whereas (ρ, φ) the extra ones($0 \leq \rho < \infty,\ 0 \leq \varphi < 2\pi$). G_{AB} is the 6D metric field, Φ^* and Φ are the complex scalars (Higgs fields). $M(> 0)$ is the 6D Planck mass. $\lambda(> 0), v_0(> 0)$ and Λ in the potential V are *vacuum parameters*. Expecting Randall-Sundrum type dimensional reduction, we assume the following form as the line element.

$$ds^2 = \mathrm{e}^{-2\sigma(\rho)} \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + \rho^2 \mathrm{e}^{-2\omega(\rho)} d\varphi^2 \quad , \\ 0 \leq \rho < \infty \quad , \quad 0 \leq \varphi < 2\pi \quad , \tag{2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. This is a natural 6D minimal-extension of the original 5D model by Randall-Sundrum [1]. Two "warp" factors, $\mathrm{e}^{-2\sigma(\rho)}$ and $\mathrm{e}^{-2\omega(\rho)}$, appear. For the fixed ρ case ($d\rho = 0$), the metric is the Weyl transformation of the product-space, the 4D Minkowski $\times$ the circle S^1. The coordinate ρ can be regarded as the *scaling* parameter. We require the periodicity in φ for $\Phi(\rho, \varphi)$ and assume the form: $\Phi = P(\rho)\mathrm{e}^{im\varphi}$ ($m = 0, \pm1, \pm2, \cdots$). The 6D classical field equations of (1) are obtained as

$$3\sigma'' - \frac{\sigma'}{\rho} + \sigma'\omega' + \omega'' - (\omega')^2 + 2\frac{\omega'}{\rho} = 2M^{-4}P'^2 \quad , \\ -16\sigma'^2 + 4\frac{\sigma'}{\rho} - 4\sigma'\omega' + 4\sigma'' = 2M^{-4}V \quad , \\ 4\sigma'' - 10\sigma'^2 = M^{-4}(P'^2 - m^2\frac{\mathrm{e}^{2\omega}}{\rho^2}P^2 + V) \quad . \tag{3}$$

We take a unit $M = 1$ in the following for simplicity. [2]

3 An Exact Solution for the No Potential Case

In order to see the structure of the solution, it is useful to consider a simple case, that is, the no potential case: $\lambda = \Lambda = 0$ ($V(\Phi^*, \Phi) = 0$). Even in this case, the scalars are still coupled with gravity. In this section we stress those properties which are shared with the general case of Sec.4. We consider the $m = 0$ (*no flux*) solution. The following exact solution of the rational function type can be obtained.

$$\sigma'(\rho) = -\frac{1}{A\rho + B},\ \omega'(\rho) = \frac{4\rho + B}{\rho(A\rho + B)},\ P(\rho)'^2 = \frac{2(2A - 5)}{(A\rho + B)^2} \quad , \tag{4}$$

[2] This is allowed by the following invariance of (1) or (3) , due to the simple dimensional counting: $\Phi \to M^2\Phi$, $(P \to M^2 P)$, $v_0 \to M^2 v_0$, $\lambda \to M^{-2}\lambda$, $\Lambda \to M^6\Lambda$, $x^\mu \to M^{-1}x^\mu$, $\rho \to M^{-1}\rho$.

where $A(\geq 5/2)$ and B are *integration constants*. A is a dimensionless constant, whereas B is a dimensional one (the dimension of length). The lower bound for A comes from the *positivity* of P'^2. This is a 2-parameters family solution (for σ', ω', and P') at this stage. Some special cases are i) $A = 5/2,\ P' = 0$; ii) $B = 0,\ -4\sigma' = \omega' = (4/A)\times\rho^{-1}, P'^2 = (2(2A-5)/A^2)\times\rho^{-2}$; iii) $A = 4,\ \omega' = \rho^{-1}$; iv) $A \to \infty,\ \sigma' = 0,\ \omega' = 0, P' = 0$. The integration constants are fixed by a required *boundary condition*.

Due to the condition $A \geq 5/2$, we do *not* find the solution which satisfies the RS-type configuration: $\sigma' \to$ const.(> 0) as $\rho \to \infty$. However, if we restrict the region of ρ as $|B| \gg A\rho$, then we have $\sigma' \sim -1/B,\ \omega' \sim (1/\rho) - (A-4)/B,\ P'^2 \sim 2(2A-5)/B^2$. This looks a RS-type metric although we *cannot* take $\rho \to \infty$.

The final solution for σ, ω and P are given as

$$\sigma(\rho) = -\frac{1}{A}\ln\frac{|A\rho+B|}{D},\ \omega(\rho) = \ln\frac{|\rho|\cdot|A\rho+B|^{-\frac{A-4}{A}}}{E^{\frac{4}{A}}},$$
$$P(\rho) = \pm\frac{\sqrt{2(2A-5)}}{A}\ln\frac{|A\rho+B|}{C} \quad , \tag{5}$$

where $C(> 0), D(> 0), E(> 0)$ are another *integration constants* with the dimension of length. They come from the symmetry of the *constant translation* ($\sigma \to \sigma +$ const.$, \omega \to \omega +$ const.$, P \to P +$ const.) in eq.(3) with $V = 0$. This final form (5) is a 5-parameters family of solutions.

The line element is given by

$$ds^2 = \left\{\frac{|A\rho+B|}{D}\right\}^{2/A}\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2 + \left\{\frac{|A\rho+B|}{E}\right\}^{2(A-4)/A}\cdot E^2 d\varphi^2 \ .$$
$$0 < \frac{2}{A} \leq \frac{4}{5} \quad , \quad -\frac{6}{5} \leq \frac{2(A-4)}{A} < 2 \quad , \tag{6}$$

For the case of $A \to \infty$ the space-time becomes locally 6D Minkowski with the deficit angle $\delta = 2\pi(1-A) = -\infty$. For the restricted region $|B| \gg A\rho$, the above metric reduces to

$$ds^2 = \mathrm{e}^{\frac{2}{B}\rho}\left\{\frac{|B|}{D}\right\}^{2/A}\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2 + \mathrm{e}^{\frac{2(A-4)}{B}\rho}\left\{\frac{|B|}{E}\right\}^{2(A-4)/A}\cdot E^2 d\varphi^2 \ . \tag{7}$$

As a form, this is a RS-type metric although we *cannot* keep it in $\rho \to \infty$ region.

The 6D Riemann scalar curvature is obtained as

$$\hat{R} = -\frac{4(2A-5)}{(A\rho+B)^2} \leq 0 \quad , \tag{8}$$

which is negative semi-definite. The absolute value goes to 0 as $\rho \to \infty$. For $A = 5/2$ (the case i) above), $\hat{R} = 0$ is valid everywhere. The metric in this case

$$
A = \frac{5}{2} \quad ,
$$
$$
ds^2 = \left\{ \frac{|\frac{5}{2}\rho + B|}{D} \right\}^{4/5} \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + \left\{ \frac{E}{|\frac{5}{2}\rho + B|} \right\}^{6/5} \cdot E^2 d\varphi^2 \ . \tag{9}
$$

For $\infty > A > 5/2$, we can see three cases: a) $B > 0$, There is *no curvature singularity*; b) $B = 0$, The curvature is singular at $\rho = 0$; c) $B < 0$, The curvature singular at $\rho = -B/A$. As for the horizon, from the condition $0 < \frac{2}{A} \leq \frac{4}{5}$, there is no horizon in the region $0 \leq \rho < \infty$. Hence, in b) and c), the singularities are naked ones.

The structure of the solution (6) is similar to the Kasner solution[12] in 1+3 D. Ref.[13] explains the cosmic string solutions in the Abelian Higgs model in 1+3 D. The two branches explained there, that is, the cosmic string branch and the Melvin branch look to have correspondence with the $A \to \infty$ case and $A = 5/2$ case respectively.

4 Pole Solution in Six Dimensions

Now we consider for the case of the spontaneous breakdown (Higgs) potential: $\lambda > 0, v_0 > 0$, and Λ is general(at this stage). The boundary condition for P should be taken as

$$
\rho \to \infty \quad , \quad P(\rho) \to +v_0 \quad , \tag{10}
$$

where we take the plus sign, using the freedom of $\Phi \leftrightarrow -\Phi$ in (3), for the asymptotic value of P. We require (based on the analogy to 5D Randall-Sundrum model[1, 2, 14]) that σ' and ω' go to constants as $\rho \to \infty$. Then we can deduce, using (3), that

$$
m = 0 \text{ (no flux)} \quad ,
$$
$$
\rho \to \infty \quad , \quad \sigma' \to \alpha \quad , \quad \omega' \to \alpha \quad , \quad \alpha = +\sqrt{\frac{-\Lambda}{10}} M^{-2} \quad . \tag{11}
$$

This result says Λ should be taken negative: $\Lambda < 0$. We choose the plus sign as the asymptotic value so that the Weyl scaling factors in (2) work as shrinking the proper distance as ρ increases. This choice guarantees the

stableness of the system. As for the behavior near $\rho = 0$(ultra-violet region), we can take, in a consistent way with (3), as

$$\begin{aligned}
&\rho \to +0 \quad , \quad \sigma' \to s\rho^a \quad , \quad \omega' \to w\rho^b \quad , \quad P \to x\rho^c \\
&a = b = 1,\ c = 0 \quad , \quad s \neq 0, w \neq 0, x \neq 0 \quad , \\
&s = \frac{\lambda}{16}(x^2 - {v_0}^2)^2 + \frac{\Lambda}{4} \quad ,
\end{aligned} \tag{12}$$

with the additional possible case: $x = 0$ with $c > 1$.

We examine the following form of solutions[11].

$$\begin{aligned}
\sigma'(\rho) &= \alpha \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!} \{\tanh(k\rho)\}^{2n+1} \quad , \\
\omega'(\rho) &= \alpha \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!} \{\tanh(k\rho)\}^{2n+1} \quad , \\
P(\rho) &= v_0 \sum_{n=0}^{\infty} \frac{e_{2n}}{(2n)!} \{\tanh(k\rho)\}^{2n} \quad ,
\end{aligned} \tag{13}$$

where c's, d's and e's are coefficient-constants to be determined. σ' and ω' are composed of *odd* powers of $\tanh(k\rho)$, whereas P is of *even* powers. They come from the behavior at the ultra-violet region (12). k is a new scale parameter which shows the *thickness* of the pole. In the following, we take $k = 1$ for simplicity. [3]

The key equation for obtaining the solution of the form (13) is the following expansion formula about $1/\rho$. (The factor $1/\rho$ appears in (3)).

$$\begin{aligned}
&\frac{1}{\rho} = \frac{2}{\tanh\,\rho} \sum_{n=0}^{\infty} \frac{s_{2n}}{(2n)!} \{\tanh\,\rho\}^{2n} \ , \quad \frac{d^{2n}}{dx^{2n}} \left(\frac{x}{\ln \frac{1+x}{1-x}} \right) \Bigg|_{x=0} \equiv s_{2n} \ , \\
&0 < \rho < \infty \ , \ s_0 = \frac{1}{2} \quad , \quad s_2 = -\frac{1}{3} \quad , \quad s_4 = -\frac{12}{5} \quad , \quad \cdots \quad ,
\end{aligned} \tag{14}$$

All coefficients are finite as shown above. We can take the limit $\rho \to \infty$ above, and see the infinite series *converges* and gives 0 : $\sum_{n=0}^{\infty} \frac{s_{2n}}{(2n)!} = 0$. We expect the coefficient-series appearing in (13) have the similar behaviors. It is a key, in the present treatment of the infinite series, that we take the expansion using powers of *tanh* ρ not those of $\mathrm{e}^{-2\rho}$. [4]

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[3] This is because eq.(3) is invariant under the change: $\rho \to k\rho,\ \lambda \to (1/k^2)\lambda,\ \Lambda \to (1/k^2)\Lambda,\ v_0 \to v_0$.

[4] We can *not* reexpress the RHS of the first equation of (14) as $1/\rho = \sum_{n=0}^{\infty} \frac{s'_{2n}}{(2n)!} \mathrm{e}^{-2n\rho}$ with *finite* coefficients. The same notice is said about (13).

We can show the above form of solutions indeed satisfy (3) if the coefficients satisfy some recursion relation[11]. The first few orders are given as

$$
\left\{
\begin{array}{c}
c_1 = \frac{1}{4\alpha}\{\frac{1}{4}\lambda {v_0}^4(1-{e_0}^2)^2+\Lambda\} \quad , \\
d_1 = -\frac{2}{3}c_1 \quad , \\
e_0 \ : \ \mbox{free parameter,}
\end{array}
\right.
$$

$$
\left\{
\begin{array}{c}
c_3 = \frac{3}{32}\frac{\lambda^2{v_0}^6}{\alpha}{e_0}^2(1-{e_0}^2)^2 + c_1(2+5\alpha c_1) \ , \\
d_3 = -\frac{4}{3}c_1(1+5\alpha c_1) \quad , \\
e_2 = -\frac{1}{4}\lambda {v_0}^2 e_0 (1-{e_0}^2) \quad .
\end{array}
\right.
\tag{15}
$$

We note one *free* parameter e_0 appears [5] and all coefficients are expressed by 4 parameters $(\lambda, v_0, \Lambda, e_0)$. The parameters, however, have 3 constraints.

$$
1 = \sum_{n=0}^{\infty}\frac{c_{2n+1}}{(2n+1)!} \quad , \quad 1 = \sum_{n=0}^{\infty}\frac{d_{2n+1}}{(2n+1)!} \quad , \quad 1 = \sum_{n=0}^{\infty}\frac{e_{2n}}{(2n)!} \quad , \tag{16}
$$

which comes from the boundary conditions (10) and (11). Therefore the present solution is a 1(=4-3) parameter family of solutions.

5 Evaluation of Coefficients and Analytic Result

We present here the results of concrete values of c's, d's and e's for an *input* value $e_0 (= P(0)/v_0) = -0.8$. We solve constraints (16) by taking the first 7 terms (up to n=6th order). The most crucial point of the present solutions is to confirm the convergence of the infinite series $\sum_{n=0}^{\infty}\frac{c_{2n+1}}{(2n+1)!}$, $\sum_{n=0}^{\infty}\frac{d_{2n+1}}{(2n+1)!}$, and $\sum_{n=0}^{\infty}\frac{e_{2n}}{(2n)!}$, which guarantees the present boundary condition. The 6th-order approximation calculation gives the vacuum parameters as

$$
v_0 = 0.9625 \quad , \quad \Lambda = -2.725 \quad , \quad \lambda = 18.375 \quad . \tag{17}
$$

[6] [7] The obtained values of the coefficients are shown in Fig.1. In the figure

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[5] This freedom corresponds to, when $\lambda = 0$, the constant translation symmetry ($P \rightarrow P + \mbox{const.}$) in eq.(3) .

[6] These values should not be confused with numerical results like Sec.6. They should, in principle, be definitely determined just like the energy eigenvalues of the hydrogen atom are fixed by the boundary condition.

[7] The constraint criterion is $(1-\sum_{n=0}^{6}\frac{c_{2n+1}}{(2n+1)!})^2 + (1-\sum_{n=0}^{6}\frac{d_{2n+1}}{(2n+1)!})^2 + (1-\sum_{n=0}^{6}\frac{e_{2n}}{(2n)!})^2 < 10^{-4}$.

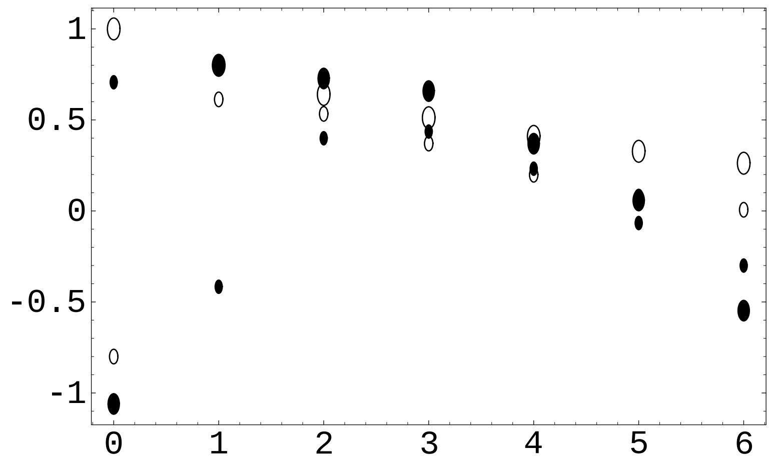

Fig.1 The values of $c_{2n+1}/(2n+1)!$(large blob),$d_{2n+1}/(2n+1)!$(small blob) ,$e_{2n}/(2n)!$(small circle). The large circles show $(0.8)^n$. ($n = 0, 1, \cdots, 6$.)

we also plot the data from the geometrical series: $1/(1-x) = 1 + x + \cdots$ at $x = 0.8$. Comparing them, we can recognize the convergence of the coefficient-series (up to this approximation order). Note that all three series are 'oscillating', which is advantageous for their convergence. Using these results, the analytical results of $P(\rho), \sigma'(\rho)$ and $\omega'(\rho)$ ((13))are shown in Fig.2. We can see comparative behaviors of two 'warp' factors (σ', ω') in the ultra violet region, $\rho \leq 2$. In particular we notice a very 'delicate' dip in the behavior of the second 'warp' factor ω' at $k\rho \sim 1$. The (6D) Riemann scalar curvature is also shown in Fig.3. It shows the curvature is positive both inside and outside of the pole, whereas negative (small absolute value) or zero between them. It is *non-singular everywhere.* There is no horizon in $0 \leq \rho < \infty$. The deficit angle at $\rho = \infty$ is 2π.

6 Numerical Results

As the coupled differential equations for $P(\rho), \sigma'(\rho)$ and $\omega'(\rho)$, the equations (3) with $m = 0$, have the standard form of the numerical analysis, that is, Runge-Kutta method. We can numerically solve (3) *without any ansatz* about the form of the solution. In this approach, the choice of the initial values with *high precision* is required. In the present case we cannot take

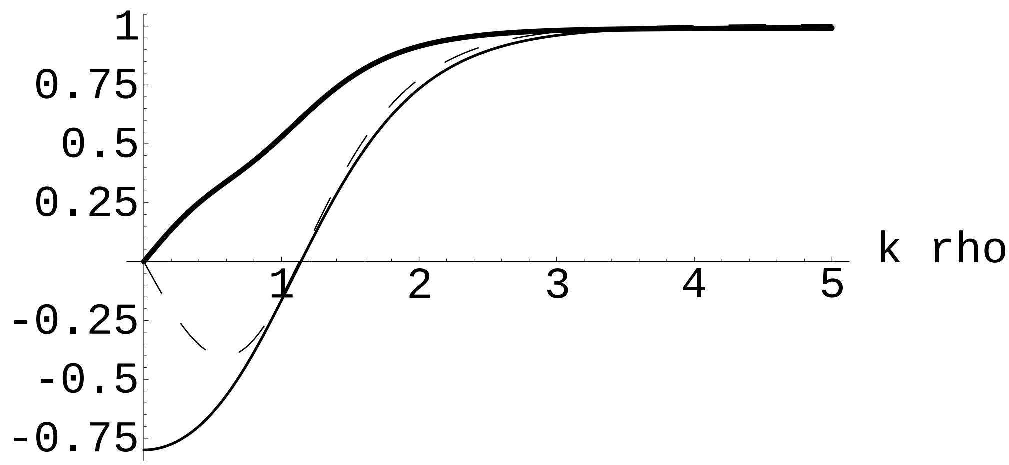

Fig.2 The analytic result of ω'/α(bold line), σ'/α(dashed line) and P/v_0(normal line). The graphs are depicted by using (13) with the 6-th order approximation. The horizontal axis is $k\rho$. We take $k = 1$.

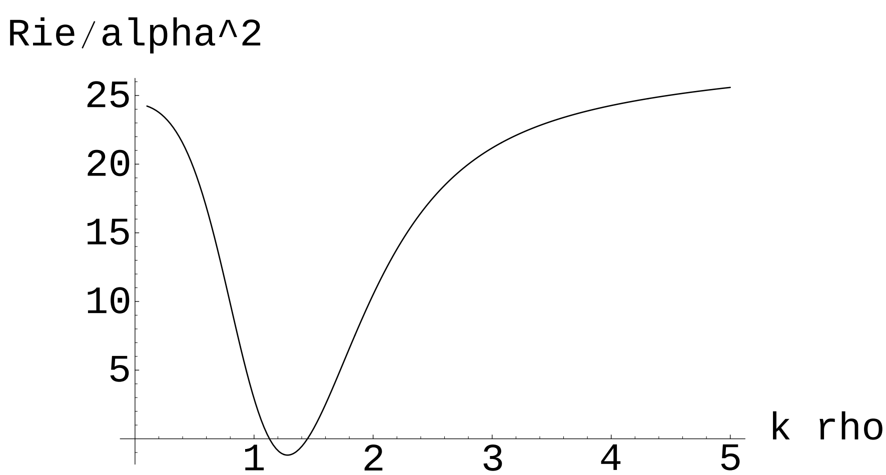

Fig.3 (6D) Riemann scalar curvature $\hat{R}/\alpha^2$ in the 6th order approximation. The horizontal axis is $k\rho$. We take $k = 1$.

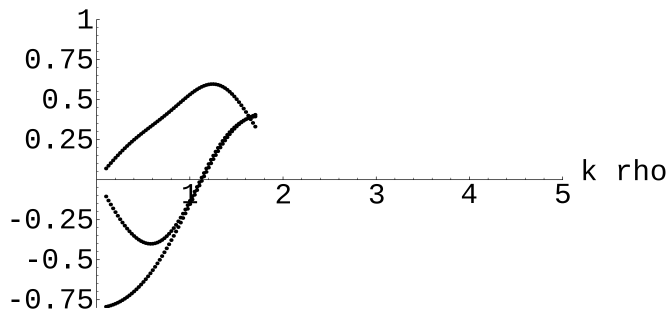

Fig.4 The numerical results for ω'/α(top), σ'/α(middle) and P/v_0(down). They are obtained by Runge-Kutta method. One step value along $k\rho$-axis is 0.025. About 65 points are plotted for each line in the figure. The initial point is $\rho = 0.1$. The horizontal axis is $k\rho$. We take $k = 1$.

$\rho = 0$ as the initial point because the appearance of the factor $1/\rho$ in (3). (Note this does not say the solution is singular at $\rho = 0$. We seek a *non-singular* solution.) We take $\rho = 0.1$ as the initial point of ρ. As for the initial values we borrow the results from Sec.5 : $P(0.1) = -0.79386$, $\sigma'(0.1) = -0.10488$, $\omega'(0.1) = 0.070046$. The result by the Runge-Kutta calculation is shown in Fig.4. It shows the analytic solution of Sec.5 is well reproduced in the ultraviolet region (small-ρ region) but not for the infrared region. We should notice that the the consistent region (with the analytic result) , $\rho \leq 1.3$, becomes definitely larger than that of the 2nd order approximation [11] , $\rho \leq 0.9$. The numerical output data stop at $\rho \sim 1.7$ with producing imaginary values. This occurs because keeping the *positivity*, $P'(\rho)^2 \geq 0$, becomes so stringent in the infrared region. The quantity becomes so small in that region and vanishes at $\rho = \infty$. We understand that further higher precision is required for the initial values in order to extend the consistent region furthermore.

7 Conclusion

We have presented the both analytical and numerical solution in the 6D reduction model. The approximation oder is $n = 6$. Compared with the previous result of $n = 2$ order one, the results improve definitely. Much evidence about the convergence of the coefficients series is obtained. An

exact solution, for the no potential case, is also obtained.

We notice rather stable behavior of the following values under the increase of the approximation order : the vacuum parameters (v_0, Λ, λ); the coefficients $(c_{2n+1}, d_{2n+1}, e_{2n} | n = 0, 1, \cdots, 6)$; the form of σ', ω' and P. This strongly indicates the result of Fig.2 is quite near the exact result.

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